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Non Linear Evolution of Dynamic Spatial Systems: Lessons from Chaos and Niche Theory

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NON LINEAR EVOLUTION OF DYNAMIC SPATIAL SYSTEMS: LESSONS FROM CHAOS AND NICHE THEORY

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ABSTRACT

Recently increasing interest in non-linear dynamic phenomena has emerged in economics and regional science, such as resilience of systems to sudden changes, adjustment processes, uncertainty and perturbations at both micro and macro levels. In the context of spatial modelling this has also provoked an increasing use of (continuous and discrete) dynamic models derived from ecology (such as May's logistic growth model which is able to generate chaotic behaviour, or prey-predator equations which are able to generate cycles or oscillating behaviour). Such models are able to depict both regular and irregular movements of spatially relevant phenomena.

This paper aims to offer a comprehensive overview of recent developments in this field, with particular attention for spatial systems. Starting from a methodological review of central concepts of chaos theory and related modelling attempts, the present paper gives a survey of possible evolutions of dynamic spatial systems known from the literature (such as the class self-organizing systems from social sciences) by investigating the connections between ecology and (spatial) economics. In particular, ecological approaches based on niche theory and competition models will be dealt with from the viewpoint of the possibility of chaotic behaviour. Some simulation results will be added as well. The paper will conclude with further reflections on new research directions which need to be developed in regional science.



1 Towards Non-linear Modelling

Despite the strong modelling traditions in various social sciences (inter alia, economics, geography, regional science, transportation science), surprisingly little attention has been given to **non-linear** models. Apparently, the large majority of model experiments and applications has taken for granted the existence of linear - and thus regular - growth processes. Admittedly linear dynamic models are certainly able to generate **unstable** solutions, but the solutions of such models are restricted to the following four regular standard types: oscillatory and stable, oscillatory and explosive, monotonous changing and stable, and monotonous changing and explosive. Such models may provide approximate replications of short - or medium - term changes, but fail to encapsulate long-term developments characterized by structure shifts of an irregular or abrupt nature. An interesting example of the increasing interest in such long-range development issues can be found in an article by Day and Walter (1990) describing the socio-economic historical evolution of societies on the basis of 'regime switches' in technology, demography and institutions; a further example in this context is given in an article by Dendrinos and Rosser (1992), where the foundations for a comprehensive dynamical theory of discontinuities in urban population size is discussed.

The interest in modelling long-run future development paths dates back to the late sixties/early seventies with the emergence of dynamic systems theory (see among others Forrester, 1968, and Meadows and Meadows, 1973). Such models - mainly simulation models - focused attention on the impact of feedback effects in dynamic systems on stable behaviour. The availability of computer software led to a high degree of popularity of such experiments, although also severe criticism arose. In contrast to linear models, such systems dynamic models were able to generate also a-periodic growth patterns.

The interest in unstable systems behaviour remained in the 1970s and 1980s, and generated new contributions in the field of structure changes (such as tensor analysis, singularity theory, bifurcation theory, etc.). Especially **catastrophe theory**¹ gained much popularity in the late seventies, mainly because it is able to describe turning points and jumps - of an asymmetric and sometimes irreversible nature - in seemingly stable systems (see for various illustrations Poston and Stewart, 1978, and Zeeman, 1977). In such systems the direction of movement and the level of threshold values determines the occurrence of catastrophes, which are essentially characterized by the fact that a dynamic system may have various equilibrium points for the same values of state variables.

Applications of catastrophe theory in the field of regional science can amongst others be found in Casti (1979), De Palma and Lefèvre (1987), Fischer and Jammerneegg (1986), Lombardo and Rabino (1983), Nijkamp and Reggiani (1988), Rosser (1991), Varian (1978), Vendrik (1980) and Wilson (1981). A weak element in catastrophe models is the fact that the identification and explanation of turning points is fraught with difficulties, so that the operational basis of catastrophe theory remained feeble. The main problem in analyzing turning points and non-linear growth patterns is the fact that such phenomena occur in a very irregular fashion, so that normal time series are often inadequate in providing sufficient statistical evidence. Thus instability can *ex post* be traced, but is very hard to predict *ex ante* (Peters, 1988). Prediction is apparently only

¹. Catastrophe is a term coined by Thom (1975) for explaining biological morphogenesis. Thom also gave a topological classification of the elementary types of catastrophes depending on the number of control variables in a dynamical system.

meaningful in case of stable evolution.

In light of these observations **chaos theory** has in recent years become a popular tool. The next section will offer a sketch of the road towards chaos.

2 Central Concepts in Dynamical Systems Preceding Chaos Theory

2.1 Structural instability

The interest in the dynamics of real-world systems is not recent, but can already be found in early analytical dynamics. Analytical dynamics aims to describe and explain transformations in complex real-world systems. It emerged about 300 years ago with the concept and calculus of differential equations developed by Newton and Leibniz. Most theories in physics (such as Einstein's relativity theory or Maxwell's electro-magnetic theory) can be represented by means of differential equations.² The use of ordinary differential equations in economics has a shorter life span than in physics and dates back to Walras in 1874. However, it was Samuelson (1947) who firmly demonstrated the scientific significance of the use of differential equations in economics. He introduced the new concept of 'dynamic analysis', as a substitute for the commonly accepted concept of 'static analysis', by pointing at Frisch's article "Propagation Problems and Impulse Problems in Dynamic Economics" (1933) (see also Medio, 1979). In this context the notion of stability in economics emerged in the late 1950s (see, for example, Allen, 1959). It should be noted at this stage that stability concepts are in economics strictly linked to dynamic equilibrium phenomena.

Roughly speaking, following Zhang (1990, p. 19) we may define equilibrium "as to the state of a dynamic system at which all variables are invariant with respect to time". Then we may assert that "an equilibrium x is stable if all nearby solutions stay nearby. It is asymptotically stable if all nearby solutions not only stay nearby, but also tend to x " (cfr. Hirsch and Smale, 1947, p. 180).³ Inversely "a system is considered to be at non equilibrium if the variables have different rates of change with respect to time. Limit cycles, aperiodic behaviour and chaos (which will be described subsequently) are examples of non-equilibria" (see again Zhang, 1990, p. 19).

In the past decades differential equations have been extensively employed as powerful analytical tools by economic researchers. However, it soon became clear that differential equations have an intrinsic limitation, namely they do usually only describe phenomena displaying regular and ordered behaviour, since their analytical solutions need differentiability conditions. Thus discontinuities or sudden and dramatic changes or jumps which often characterize real-world phenomena cannot satisfactorily be analyzed by differential equations.

A second shortcoming is the following: it is often impossible to find the explicit solution of non-linear differential equations (i.e., equations where the coefficients of the

². Differential equations can in general be formalized as follows:

$$\dot{x}_i = f_i(x_1, x_2, \dots, x_n; \alpha_1, \alpha_2, \dots, \alpha_m) \quad i = 1, 2, \dots, n$$

where the symbol $\dot{}$ indicates dx/dt , (i.e. the derivative of function x with respect to the variable t , with t being time), and where α are relevant parameters.

³. For the classification of different kinds of (in)stability related to a general system of two differential equations, see also Annex 6A in Nijkamp and Reggiani, 1992a.

state variables are not constant), unless one resorts to approximations via linear equations. But this means again that unexpected discontinuities, a multiplicity of (dis)equilibria as well as a multiplicity of bifurcations and catastrophe behaviour are missed out. More modern approaches try to overcome the above mentioned limitations by analyzing not just the properties of particular solutions of differential equations, but the sensitivity of solutions to perturbations (or their stability), in other words, their structural (in)stability.

In this framework we can define a system as 'structurally unstable' when even small perturbations in functional forms change the qualitative properties of the system. In other words, structural instability means the possible existence of significant qualitative changes in the behaviour of the system (i.e., in the state variables) which are closely connected with bifurcation and catastrophe phenomena that (can) occur when the parameters values (i.e., the control variables) are changing (see for a brief review on bifurcation theory and catastrophe theory applied to spatial-economic patterns among others Day, 1985, Dendrinos and Mullally, 1985, and Zhang, 1991).

It is well known from the literature that a simple bifurcation can transform the equilibrium points from one state (e.g., stable) to another one (e.g., unstable) (or vice versa). An example of such structural instability is provided by a prey-predator system, since a bifurcation (depending on the parameter values) can transform the equilibrium point from a stable 'focus' into an unstable 'focus' via a 'centre' (for a precise description we refer to Nijkamp and Reggiani, 1992a, Ch. 8).

In conclusion, it is possible to analyze, by means of the concept of structural stability, various types of complex dynamics in real world systems, such as the evolution of cities (see Mees, 1975), regional development processes (Anderson and Batten, 1988, and Wilson, 1981), dynamic choices in spatial systems (Fischer et al., 1990) or economic equilibrium analysis (Turner, 1980).

It should be added that a further analysis of dynamics in complex systems should focus on the time-dependent behaviour of the system (compared to the phase portrait or the bifurcation diagrams plotting the state variable values against the parameter values). In this context the identification of (a)periodic motions (or oscillations in the state variables) results as an interesting outcome.

2.2 (A)periodic motions in behaviour

Periodic or cyclic solutions are very common in economics (see, for example, the wealth of business cycle literature reviewed by Lorenz, 1989, Kelsey, 1988, Puu, 1989, and Zhang, 1991) as well as in ecology (see, e.g. Haken, 1983a and 1983b), and recently also in geography and regional science (see, e.g., Dendrinos and Mullally, 1985, Nijkamp, 1987, and Nijkamp and Reggiani, 1990).

From a mathematical point of view periodic solutions are strictly connected with the existence of closed orbits and limit cycles. In the first case states are repeated from one orbit to the next; in the second one this does not hold, but then the orbits are asymptotically close to a closed orbit. In other words, a limit cycle is a closed orbit (emerging as an equilibrium solution from two-dimensional differential equations) which is also an attractor (i.e., a bounded region towards which every trajectory of a dynamic system tends to move). Thus a limit cycle is "closed so that a point moving along the curve will return to its starting position at fixed time intervals and thus execute periodic motion" (cf. Zhang, 1990, p. 207).

It should be noted that there is also the possibility of so-called aperiodic or

'chaotic' oscillations when the amplitude and period vary without any trace of a recognizable pattern (see also next subsections).⁴

A method to find limit cycles in the two-dimensional plane is provided by the Poincaré-Bendixson theorem (see e.g. Haken, 1983a). The Poincaré-Bendixson theorem conjectures in particular the existence of two kinds of attractors for a two-dimensional flow: (1) stable fixed points (see for a classification of fixed points, Nijkamp and Reggiani, 1992a) and (2) periodic solutions or limit cycles. Consequently, the chaotic motion, which is associated with the so-called **strange attractor**, as we will show hereafter, is not possible for two-dimensional flows (see also Lichtenberg and Lieberman, 1983).

Now, it should be noted that the change from a fixed point into a limit cycle (if a single parameter varies) is a phenomenon known as **Hopf bifurcation**. It is interesting to note that in an at least three-dimensional system the Hopf bifurcation can be associated with chaotic behaviour (see Marsden and McCracken, 1976). In this framework Newhouse et al. (1978) have shown that after three Hopf bifurcations regular motion becomes highly unstable in favour of motion to a **strange attractor** (defined here as a bounded region of phase space in which initially close trajectories separate exponentially such that the motion becomes chaotic). It is thus clear that central in the analysis of dynamical systems is also the idea of an attractor, i.e. a set of states in the state space of the system which attracts all trajectories emanating from neighbouring points.

2.3 (Strange) attractors

Given the above definitions of attractors, it is clear that the attractor does not describe change over time. Indeed, for many parameter values of a dynamical system its attractors (if they exist) are regular objects described by equilibrium points or closed curves. For other parameter values they may have an extremely complex structure. Such attractors are called "strange", a term coined by Ruelle and Takens (1971) in order to indicate an "exponential separation of orbits" (see Day, 1985, and Eckmann and Ruelle, 1985). Strange attractors are strictly connected to chaotic motion (see Annex 6B in Nijkamp and Reggiani, 1992a, for a review of major classes of strange attractors). In particular, in 1975 Li and Yorke christened 'chaotic' a system with strange attractors or a dynamic situation exhibiting aperiodic - though bounded - trajectories.

A more strict definition of a strange attractor is provided by Schuster (1988, pp. 105-106) as follows:

- " a) It is an attractor, i.e., a bounded region of phase space to which all sufficiently close trajectories from the so-called basin of attraction are attracted asymptotically for long enough times. We note that the basin of attraction can have a very complicated structure. Furthermore, the attractor itself should be indecomposable; i.e., the trajectory should visit every point in the attractor in the course of time. A collection of isolated fixed points is no single attractor.
- b) The property which makes the attractor strange is the sensitive dependence on the initial conditions; i.e, despite the contraction in volume, lengths need not shrink in all direction, and points which are arbitrarily close

⁴ An interesting review on illustrations of (a) periodic motions with a special focus on technological change, business cycle and economic growth and competition can be found in Batten, Casti and Johansson (1987).

initially, become exponentially separated at the attractor for sufficiently long times."

Thus strange attractors (together with the other attracting regions in phase such as fixed points and limit cycles) are pertinent to **dissipative**⁵ systems for which volume elements in phase space shrink with increasing time.

Strange attractors are also connected with the notion of **fractal dimensions**. Fractal is a term coined by Mandelbrot (1977), in order to illustrate that similar structures may repeat themselves at higher orders or dimensions. Essentially, a fractal set is a set having the property of being invariant at different scales (self-similarity and irregularity property) and having a non-integer, fractional dimension (for an application of fractal geometry to urban structure see, e.g., Batty and Longley, 1986, and Frankhauser, 1991). Therefore, the notion of fractals only refers to the geometry of attractors (see also Mandelbrot, 1977, and Peitgen and Richter, 1986).

Since the notion of strange attractors usually refers to the dynamics of the attractors (and not just to their geometry), strange attractors need not have a fractal structure and attractors with a fractal structure need not be chaotic (see, e.g., Holden and Muhamad, 1986).

The concept of 'chaos' results straightforward from the previous basic concepts. Its significance rests essentially on the possibility of studying a variety of natural and social science based models where non-differentiability is typical.

2.4 Chaos

Chaos has in recent years become the new paradigm for studying non-linear dynamic systems. During the 1980s chaos theory "received widespread prominence and some scientists have even placed it alongside two other great revolutions of physical theory in the twentieth century - relativity and quantum mechanics. While those theories challenged the Newtonian system of dynamics, chaos has questioned traditional beliefs from within the Newtonian framework" (Crilly, 1991, p. 193).

The interesting characteristic feature of chaos theory is that it addresses essentially the (in)stability of **deterministic, non-linear dynamic systems** which are able to produce **complex motions** of such nature that they are sometimes seemingly random. In particular such systems incorporate the property that small uncertainties may grow exponentially (although all time paths are bounded), leading to a broad spectrum of different trajectories in the long run, so that precise or plausible predictions are - under certain conditions (see below) - almost impossible. This phenomenon, which is typical of chaotic dynamics, is known as **sensitivity to initial conditions**. Already at the beginning of this century it was recognized by Poincaré that "it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon" (Poincaré, 1913, p. 397).

In particular in a chaos system two sets of conditions, which initially are very close, together may become exponentially separated⁶ giving rise to widely different states in the

⁵ However one can find 'chaotic' (in the sense of irregular) regions also in conservative systems, but they are not attractors.

⁶ This exponential separation of adjacent points can be measured by means of a positive Liapunov exponent. For its analytical formulation see Schuster (1988) and for the calculus of algorithms see Wolf et al. (1988).

long run⁷. Thus, it follows from the previous remarks that predictability of long term behaviour may become problematic for those non-linear systems which incorporate chaotic dynamics.

Consequently, the new logic which has emerged in the field of non-linear dynamics by the introduction of the theory of chaos has also an interesting psychological appeal; model builders need not necessarily be blamed any more for false predictions, as errors in predictions may be a result of the system's complexity, as can be demonstrated by examining more carefully the properties of the underlying non-linear dynamic model structure. However, even though from a behavioural point of view chaos is sometimes identified with 'dynamical stochasticity', 'self-generated noise', or 'intrinsic stochasticity' (see, for example, Hao, 1984), from a mathematical point of view, there is still some uncertainty regarding the precise definition of chaos: "clearly there is still a lot of 'chaos' in chaos theory" (Rosser, 1991, p. 30). On the one hand, chaos is identified by many authors as aperiodic behaviour (see, for example, Guckenheimer, 1979 and Nusse, 1987), starting from Li and Yorke (1975) who, in their well-known theorem 'Period Three Implies Chaos', identify chaos -for any continuous mapping of a one-dimensional interval onto itself - with the existence of cycles of all orders and a scrambled set in which all trajectories are non-periodic and asymptotically unstable. In this spirit we recognize also the 'Feigenbaum route' and the 'intermittency route' to chaos. Feigenbaum (1978) in particular discovered the phenomenon of a cascade of bifurcations, each leading to period doubling sequences as a system approaches chaos. Manneville and Pomeau (1979) found that chaos can be intermittent, i.e., chaos can emerge and then be replaced by a new zone of stable equilibrium.

On the other hand, chaos can be identified with the existence of strange attractors, like in the 'Ruelle-Takens-Newhouse route' (Ruelle and Takens, 1971, and Newhouse et al., 1978). These authors showed in particular that after three Hopf bifurcations, it is 'likely' that regular motion becomes highly unstable in favour of motion to a strange attractor.

However, Eckmann and Ruelle (1985) argued that it is the sensitive dependence on initial conditions which is the true meaning of chaos, as indicated by the presence of Liapunov exponents (see footnote 5). In this context it appears fundamental to have a characterization of chaos by measuring the degree to which a dynamical system is chaotic. The K Kolmogorov entropy (see e.g. Schuster, 1988) is probably the most important approach in this respect, since it is proportional to the rate at which information about the state of the dynamical system is lost in the course of time. In other words, K is the mean rate at which two distinct, but empirically indistinguishable starting points, produce - as time passes - trajectories which are distinguishable. It is clear that K is connected with a positive Liapunov exponent. In particular the latter index quantifies the stretching and contracting in various directions, while the former measures an aggregate of the stretching.

Thus it is a more 'rigorous' and computationally easier approach to test chaos by means of a positive K or a positive Liapunov exponent instead of using the Li-Yorke theorem, the Feigenbaum route or the other routes to chaos.

It should be added, however, that in an empirical context Liapunov exponents do

⁷ This is the so-called 'butterfly effect' (Lorenz, 1963): the single flap of a butterfly's wings might theoretically alter the initial conditions of weather systems giving rise to completely different weather scenarios in the long term.

not seem to be sufficient in providing evidence for chaos (see Brock, 1986). In particular Brock proposed - by pointing at a striking property of chaotic equations (i.e., invariance to linear transformations) - a residual test for economic time-series, based on the following method. If one carries out a linear transformation of chaotic data, then both the original and the transformed data should have the same Liapunov exponent as well as the same correlation dimension⁸. Thus if these indices would appear to be substantially different, then the hypothesis of a deterministic law should be suspicious (see also Frank and Stengos, 1988). This test addresses also the issue of random disturbances in economic activities. There are many independent sources of noise which affect economic data. It is however possible that some of the observed noise is not 'extrinsic noise', independent of the economic system, but 'intrinsic noise' generated by chaotic dynamics of the system itself (see Kelsey, 1988).

It is interesting to note in this context that it has recently been shown that chaotic behaviour is less sensitive to noise than periodic orbits (see again Kelsey, 1988). Because of this low sensitivity of chaotic equations to random errors, it turns out to be difficult to identify the nature of random disturbances in time series (see also Section 4).

In conclusion, chaos is not regarded any more as a peculiar theoretical possibility, but also as a practical issue for empirical research. Examples of chaotic empirical phenomena can be found, inter alia, in solar pulsation (Kurths and Herzel, 1986), cardiac cells (Glass et al., 1983), long-term climatic change (Nicolis, 1986), measles epidemics (Schaffer and Kot, 1985), hydrodynamic turbulence (Swinney, 1983), and biological and physiological systems such as nephrons, neural and metabolic-networks (Degn et al., 1987). Interest in economics is straightforward. It will be dealt with in the next sections. Thus chaos "has proved to provide a fruitful approach to organizing one's thoughts concerning many observed phenomena" (Frank and Stengos, 1988, p. 104).

3 Modelling Chaos

Already in 1976 May argued in his seminal contribution "Simple Mathematical Models with Very Complicated Dynamics": "Not only in research, but also in the everyday world of politics and economics, we would all be better off if more people realized that simple non-linear systems do not necessarily possess simple dynamical properties" (p. 157). In this article, May also investigated how apparently erratic fluctuations in census data (e.g. for an animal population) may originate from a rigidly deterministic population growth relationship such as the following first-order difference equation:

$$x_{t+1} = F(x_t) \quad (1)$$

where $F(x)$ is a non-linear function. In the literature, we observe often more specifically the following 'logistic' difference equation⁹:

$$x_{t+1} = a x_t (1-x_t) \quad (2)$$

In equation (2) x represents the biological population while the parameter a is the growth parameter reflecting the maximum per capita rate of increase of the time-dependent

⁸. The concept of correlation dimension was introduced by Grassberger and Procaccia (1983a,b). The correlation dimension measures the ratio between the spatial correlation (between all points on the attractor - for a given radius r) and r (see for details Lorenz, 1989).

⁹. It is interesting to point at a note by Frank and Stengos (1988) who argue that (2) is an example of a semi-deterministic process rather than a deterministic process: "A process is said to be deterministic if its entire future course and its entire past are uniquely determined by its state at the present instant of time. A semi-deterministic process has its future determined by its present. The past may or may not be determined by the present" (p. 108).

variable x . The logistic map (2) requires for its existence that $0 \leq x \leq 1$ and $0 < a < 4$. Equation (2) exhibits fixed points (or equilibrium values, i.e., values that do not change when the mapping is iterated) as well as bifurcations of fixed points (see Figure 1).

Figure 1 about here

Model (2) has been extensively discussed by May and subsequently by many other authors, for instance, Baker and Gollub (1990), Baumol and Benhabib (1989), Frank and Stengos (1988) and Kelsey (1988), so that we will not discuss here in the detail the possible evolutionary patterns of x . We will just underline that for $a > a^* = 3.824..$ a cycle of period 3 appears (e.g., in biology a population value which reiterates every third generation), beyond which there are cycles in every integer period, as well as an uncountable number of aperiodic trajectories; in other words, according to the above mentioned statement of Li and Yorke, this is a typical example of a chaotic region (see again Figure 1).

It is interesting to note in this context that subsequently Feigenbaum showed that chaos can occur in all first-order difference equations of type (1) in which $F(x)$ has (after a proper rescaling of x) only a single maximum in the unit interval $0 \leq x \leq 1$. Moreover, chaos can appear in (2) at a value of a' preceding a^* , i.e. when the number of fixed points after doubling at distinct, increasing values of a becomes infinite (see for details Schuster, 1988) ending up in the so-called Feigenbaum attractor.¹⁰

It is a surprising fact that already in 1838 Verhulst introduced equation (2) for simulating the growth of a population in a closed area. Further applications of (2) to economics include, amongst others, macroeconomic models (e.g. Stutzer, 1980, and Day, 1982), models of rational consumption (see e.g., Benhabib and Day, 1981), models of overlapping generations (see Benhabib and Day, 1982, and Grandmont, 1985). Indeed, non-linearities of the type of equation (2) including a saturation effect have a structure often found in (spatial and industrial) economics. However, such equations - often used in the analysis of dynamic economic systems - have posed the question regarding the relevance and possibility of using discrete time periods in applied economic research. Firstly, the hypothesis of a fixed delay is often considered as a shortcoming, since it implies the presence of rigid discontinuities in technology or in the economic agents' memory and expectations. Secondly, it may be questioned whether the results obtained by applications of (2) capture plausible indigenous features of real economic systems or whether they are only of mathematical interest. An interesting approach in this debate is the one provided by Medio (1989) who shows how a continuous approximation of (2) - if carried out correctly (i.e. by realizing that a discrete model is an infinitely dimensional one) - yields in a continuous setting essentially the same qualitative results as those obtained in a discrete setting.

From a mathematical point of view, chaos can also arise in higher-order difference equations, in systems with more than one variable, and in systems of differential equations of dimension of at least three¹¹. Two well-known examples are the Hénon map and the Lorenz equations for these two cases, respectively (see, for an illustration, Annex

¹⁰ The Feigenbaum attractor is not a strange attractor, because the related Liapunov exponent is still zero. However, it should be noted that for the values of $a > a'$ the Liapunov exponent becomes positive which points at the existence of the first chaotic region.

¹¹ The fact that chaos cannot arise in one- or two-dimensional systems of differential equations can be explained by the Poincaré-Bendixon theorem (see Section 2.2) which demonstrates that any attractor of a two-dimensional system of differential equations is either a fixed point or a limit cycle.

6 B in Nijkamp and Reggiani, 1992a)

Since the main characteristic of these systems is to be dissipative (see Section 2.3), a new issue emerges here, viz. the classification of economic systems in dissipative (where volume contracts) or conservative (where volume is preserved) systems. Most physical systems are dissipative due to friction forces. However, in economics we do not yet have a well established economic concept corresponding to the above energy or friction concepts.

4 Chaos Theory in Economics

A characteristic feature of chaos models is their extreme sensitivity to initial conditions and values of critical parameters. In this context it can be demonstrated that measurement errors and specification errors may produce the same results for the evolution of a dynamic system (cf. Crutchfield et al., 1984). Such complex dynamics is often prevailing in economic systems, especially at a micro level and in short time spans. It is clear that aggregate phenomena (e.g., GNP, employment) measured in the long run do not easily exhibit drastic and unexpected changes, because diversity smoothing amendments and feedback mechanisms will level out peaks and valleys. Nevertheless, various attempts have been made in the past decade to show the significance of the chaos principle for economic theorizing and applied research. Especially in growth theory and the theory of financial markets many applications can be found. Interesting surveys can be found amongst others in Baumol and Benhabib (1989), Baumol and Quandt (1985), Benhabib and Day (1981,1982), Boldrin (1988), Brock (1986), Chen (1988), Grandmont (1986), Kelsey (1988), Lorenz (1989), Nijkamp and Reggiani (1992a), Radzicki (1990), Rosser (1991), and Scheinkman (1990).

The number of applications in economics is vast and the following list gives only a sketch of the wealth of current research areas:

- economic growth theory, with a particular emphasis on business cycles (Balducci et al., 1984, Brock and Sayer, 1988, Day, 1982, Funke, 1987, Grandmont, 1985, Guckenheimer et al., 1977, Hommes et al., 1990);
- theory of structural economic change, with particular emphasis on the emergence and existence of long waves (Nijkamp, 1987, Rasmussen et al., 1985, Sterman, 1985).
- innovation theory, with particular emphasis on R&D behaviour (Baumol and Wolff, 1983, Nijkamp et al., 1991).
- theory of economic competition, with particular emphasis on limited competition and game theory (Albin, 1987, Dana and Montrucchio, 1986, Deneckere and Pelikan, 1986, Rand, 1978, Ricci, 1985);
- theory of economic equilibrium, with particular emphasis on growth and trade (Hommes and Nusse, 1989, Lorenz, 1987, and Nusse and Hommes, 1990).

Thus there is apparently a wide variety of chaos models in economics. Brock and Dechert (1987, 1991) suggest various channels by which chaotic dynamics might appear, such as the introduction of a heavily discounted future, the simultaneous use of increasing returns and externalities, the introduction of overlapping generations models, the existence of lagged effects in consumption on technology, the use of exogenous forcing functions and so forth.

By way of illustration we will brief discuss here a simple model developed by Nijkamp (1987) for analyzing long-term (spatial) economic fluctuations. In a simple arrow scheme this model can be presented as follows:

Figure 2 about here

This system can be formalized by using a dynamic Cobb-Douglas type of production function with a declining marginal product of capital caused by limiting factors causing congestion (i.e., a 'ceiling and floors' model). Thus, capital productivity depends on the difference between the ceiling (upper capacity limit) and the actual production level.

Then the resulting mathematical shape of this model is:

$$DY_t = \alpha (1 - \kappa Y_{t-1} / Y_{\max}) Y_{t-1} + \beta Y_{t-1} \quad (3)$$

where $DY_t = Y_t - Y_{t-1}$.

This model is essentially a variant of the well-known May model. Outside the equilibrium solutions this model has a complex trajectory which is determined by the growth rate α and the starting values of the variables. In case of a moderate growth rate, this model appears to generate a stable logistic growth pattern. However, if the growth rate α becomes extremely high, chaotic patterns appear to emerge.

In a subsequent paper (Nijkamp et al., 1991) the previous model had been extended toward a Harrod type growth model by incorporating also investment and savings behaviour. Next R&D investments are endogenized, by assuming that the growth path of income, consumption and investment is co-determined by R&D investments. By imposing next the condition of a declining marginal efficiency of R&D expenditures and finally even of a saturation level, one faces the possibility of diseconomies of scale. The (maximum) saturation level plays the same role as Y_{\max} in the previous equation. By means of various simulation experiments in both a single region and a multi-region system the authors were able to analyze the complex dynamic behaviour of an evolutionary (spatial) economic system, displaying the possibility of both stable and chaotic dynamics.

The previous observations and discussions have made clear that chaos concepts offer an appealing research area for theoretical economic issues of stability, bifurcations or catastrophes in a complex dynamic system. However, Brock (1989) was right in pointing out that "these papers do not demonstrate that values of model parameters needed to generate chaotic equilibria can be found that are consistent with empirical studies. To put it another way, no one has estimated a class of models that allow chaotic behaviour and found estimated model parameters consistent with chaotic dynamics" (p. 428-429).

Thus there is a need for a more rigorous statistical test on chaos in empirical data. One of the most widely used tests is the so-called BDS test (see Brock et al., 1987), which attempts to identify whether a given data set has chaotic properties (rather than building a chaos model which would use these data as an input). It turns out that - although various non-linearities may be identified - the statistical inference theory for chaotic and non-linear dynamic does not easily and plausibly demonstrate the existence of chaos in macro time series. Besides, the econometrics of chaos is entirely underdeveloped, so that there is still a formidable research task in this area.

5 Relevance of Chaos in Regional Economics: Applications

5.1 Introduction

"There are traditionally two contrasted viewpoints on the working of dynamic socio-economic systems. The 'fundamentalist' viewpoint claims that such systems are inherently stable and the origins of observed fluctuations is to be found exclusively in

exogenous variations of the 'fundamentals'. Another ('Keynesian') viewpoint claims that a significant part of observed fluctuations may be due to endogenous factors, in particular to volatile expectations" (Grandmont, 1991, p. 293).

In this section we will show that recent advancements in a spatial-economic context support the second 'endogenous' viewpoint. For this purpose we will describe here four main areas in which chaos theory has been analyzed in spatial-economic systems:

- urban systems
- transport systems
- migration systems
- industrial / production systems

Clearly, this is not an exhaustive list, but in any case it offers a reasonably representative overview of recent developments.

5.2 Urban systems

After the urban dynamics work by Forrester (1968), the first study related to this research area has been undertaken by Dendrinos in 1984 who used a May-type equation for modelling urban macro-dynamics. More specifically he adopted the following form:

$$y(t+1) = Ay(t)[B - y(t)] \quad (4)$$

where $y(t) \leq B$ represents the population and $A, B > 0$ are relevant parameters. The author showed that formulation (4) was able to satisfactorily replicate urban aggregate dynamics in the U.S. for the period 1890-1980. He observed in particular that the size of urban areas always affects (inversely) the amplitude or the number of the oscillations required to reach a steady state.

Subsequently, following this research line, Dendrinos and Sonis (1988) investigated regional relative population dynamics on the basis of a one-dimensional discrete map¹². Their analysis showed the influence of the level of spatial disaggregation on the analysis of dynamic instability; in particular, they were able to demonstrate that for the U.S. population the qualitative dynamics of the U.S. regional paths differ significantly when different levels of spatial disaggregation are employed.

In another article (Dendrinos and Sonis, 1987) the authors explore the onset (i.e., the initial point) of turbulence in discrete relative multiple spatial dynamics by demonstrating - for a three location problem - the existence of local (when two variables out of three are in an oscillatory motion, in particular in a stable two-period cycle) and partial (when the cycles are period) turbulence.

In recent works (Dendrinos, 1991, Dendrinos and Sonis, 1990 and Sonis, 1990), this topic was generalized by presenting a specification of a universal map of relative population dynamics in discrete space as follows:

$$x_i(t+1) = F_i(t) / \sum_j F_j(t) \quad F_i > 0 \quad i, j = 1, 2, \dots, I \quad (5)$$

which allocates in period $t+1$ relative distributions of a population over a set of heterogeneous locations I on the basis of comparative locational advantages depicted by functions $F_i(t)$ in a previous time period t . Then, depending on the significance of $F_i(t)$,

¹². We recall here that a discrete map is a mathematical relationship that allows the next value x_{t+1} of a quantity x to be obtained from its present value x_t .

it could be shown that (5) may provide interesting new insights into spatial population dynamics including stability, periodicity and period doubling cascades, quasi-periodicity and -aperiodic (chaotic) motions.

The non-linear evolution of a city characterized - during a certain phase of its life cycle - by a structural decline was analyzed by Nijkamp and Reggiani (1992a). In particular, the authors employ three meso-behavioural equations based on the following three key variables:

$$\begin{aligned} x &= \text{city size} \\ y &= \text{employment} \\ z &= \text{urban attractiveness.} \end{aligned} \quad (6)$$

They assume, for the evolution of (6), a general dynamic urban system (for a diagrammatic representation in Stella format see Figure 3) which appears to be characterized by Lorenz equations, although - in contrast to the Lorenz model - their model has seven structure parameters instead of three.

In particular they show - by means of simulation experiments - that this generalized system is - under certain conditions - essentially a structurally unstable system, very sensitive to small changes in the parameters and in the initial conditions; hence - with the property of displaying oscillating behaviour - chaotic patterns may arise, depending on the parameter values as well as on the initial size of urban areas.

Along the research line developed by Nijkamp and Reggiani, Zhang (1991) uses also a Lorenz system in the context of urban

Figure 3 about here

dynamics. In particular he considers the following three variables:

$$\begin{aligned} x &= \text{output of the urban system} \\ y &= \text{number of residents} \\ z &= \text{land rent} \end{aligned} \quad (7)$$

and then introduces nine parameters. By using transformations of the parameters as well as of the variables, Zhang finally shows the equivalence between his model and the Lorenz model. Thus also this model may exhibit chaos.

The main conclusion from this subsection is that **models based on the theory of chaos do not ensure that chaos will actually occur**; however, we may expect the emergence of irregular dynamic behaviour, depending on initial conditions and on critical parameter values.

An interesting remark concerns the type of chaotic models which are realistic in an economic-geographic context. For example, the logistic equation of the May-type is very simple in nature but it does not contain feedback effects that are typical of real world phenomena. In this respect the generalized Lorenz model, applied to urban decline, is richer in scope. However, owing to feedback effects that often stabilize a system, it is not so easy to find critical values of the parameters leading to a 'mathematical' route to chaos with feasible values for the variables concerned.

In this context it is worth mentioning related stream of research which has focused on fractal aspects of agglomerations (see, e.g., Batty, 1991, Batty and Longley, 1985, Frankhauser, 1991). Since in general such fractal structures were not deliberately designed, the conclusion is warranted that the fractality of agglomerations follow a certain 'order principle', despite their irregularity. Thus also in this context we may conjecture the same results as deduced from the previous chaos studies: the possibility of chaotic or irregular behaviour always exists in a non-linear dynamic system, even though it may be embedded in a deterministic ordered structure.

5.3 Transport systems

Turbulent transport dynamics has been investigated by Dendrinos (1988) who incorporates a (gravitational) spatial interaction model in a Volterra-Lotka spatial population dynamics model. He shows that the amplitude of the ecologically driven stock dynamics constrains the magnitude of oscillations in the dynamics of spatial gravitational flow interactions. Furthermore, the author considers a spatial interaction problem (i.e., a congested transportation framework) by showing that the discrete iterative dynamic equilibria are either stable attractors or stable two-period cycles.

The dynamics of choice itself in a transport system (choice among modes, routes, destinations, etc.) has next been analyzed by Nijkamp and Reggiani (1992a) who demonstrate the compatibility between dynamic logit models (belonging to the class of discrete choice models) and the May equation. Since a logit model is formally equivalent to a spatial interaction model this interesting result shows also the possibility of chaotic behaviour for both these dynamic models (see, for example, the dynamic entropy model developed by Roy (1992) for planning public facilities). Then under certain conditions for the utility function, a dynamic logit model (in its difference version) can be shown to exhibit - in principle - chaotic behaviour. Moreover, the impact of multiple lags in a dynamic discrete-time choice context was studied by the same authors. They show that n delays lead to complex behaviour with likely the emergence of a strange attractor in $n+1$ dimensions. However it also turns out that in case of a high value of n a) the related 'chaotic' pattern shows an 'ordered' structure; b) the past has no more impact on the choice process.

5.4 Migration systems

Migration systems are another example of rapidly changing evolutionary paths. Reiner, et al., (1986) have shown that migratory systems, modelled according to the stochastic equation of motion (the so-called **master equation**), can "provide another example, to which the concepts of strange attractors and deterministic chaos fully apply under certain trend parameter conditions" (Reiner et al., 1986, p. 305). They show that an endogenous migratory system exhibits chaotic behaviour only under very strong conditions. Under usual and less strong conditions regular behaviour of migration systems are more likely to emerge.

Along this research line Mosekilde et al. (1988) used a similar migratory model for illustrating the concepts of attractors, bifurcations, Poincaré sections¹³, and return maps.

Then in a further contribution Sturis and Mosekilde (1988) show the existence of strange attractors in a four-dimensional migratory system. In their study the inclination to move is utilized as a bifurcation parameter.

5.5 Industrial / Production systems

Finally we will discuss here some contributions to chaos modelling in industrial and production systems. White (1985) has investigated the conditions under which chaotic behaviour arises in an industrial system. In particular, he models the growth (or decline) of each sector in each centre by using difference equations of the May type. White's simulation results show that the value of the growth rate for which chaotic behaviour

¹³. The Poincaré section is the cross section of the orbits of one dimension lower than the space of the orbits.

appears is inversely related to the number of centres. Furthermore, the author calls attention for different degrees of chaos for the equations considered.

The Lorenz system (again in a generalized form with four parameters) has also been applied by Dendrinos (1986) to regional industrial employment evolution. The results of this model show that the trajectories converge towards periodic orbits, although such orbits may not always be well defined. Moreover, the origin of this system might be an attractor - instead of a repulsor like in the Lorenz model - implying an extinction of the socio-spatial system at hand. The conclusion seems that unexpected behaviour may exist depending on fluctuations in the model parameters induced from exogenous changes.

5.6 Concluding remarks

The previous examples show the rich variety of chaos research in spatial economic systems. It is also clear that much research still has to be undertaken in order to fully understand the chaotic possibilities in spatial economic and geographic systems.

In this context much emphasis has to be put on the analysis of the speed of change of the parameters since some critical parameter values can lead to a chaotic movement with unfeasible values for some economic variables. Also the predictive value of such chaotic models has to be questioned, since - despite their deterministic nature - they may lead to unexpected results.

It is evident that the lack of available data in spatial economics often forces researchers to focus on theoretical models that are able to predict chaos as a logical outcome of 'reasonable' hypotheses instead of finding statistical evidence of chaotic movements from time series. However, also from a theoretical viewpoint the mathematical conditions (mainly based on the range of the parameter values) for reaching chaos appear very strict. Thus following Sterman (1988), a first critical issue is that "the significance of the results hinges in a large measure on whether the chaotic regimes lie in the realistic region of parameter space or whether they are mathematical curiosities never observed in a real system" (p.148). A second important issue concerns the question of the overall stability of a system's model, if one of the subsystems is governed by chaotic motion. Thus the question is: how robust is a whole system, if one of its constituents exhibits chaotic behaviour? For this purpose we will give in the next section an illustration of a model, in which the impact of the logistic growth of a May-type equation will be studied in a broader system, by referring to the competing dynamics of an ecologically-based model.

6 Impact of Chaos in a Competing Ecologically-Based System

6.1 Ecologically-based models in economics

In the present section we will address chaos issues in the context of spatial competition and interaction by using principles from dynamic ecology. Social sciences appear to orient themselves increasingly towards the methodology of natural sciences. The analysis of the evolution of dynamic systems is, for instance, more and more based on concepts from ecology. In this context the potential of using the formalism of mathematical ecology in economics is advocated by an increasing number of scholars. We may refer here to Samuelson (1971) who attempted already more than 20 years ago to construct a unified economic-ecological theory. But it is noteworthy that already in 1932 Lotka claimed that "economic competition is only a special form of more general

phenomenon of biological competition". However, the real initiator of this dialogue between economics and ecology was essentially Malthus (1798) with his principle (and model) of population dynamics and saturation, including his scientific influence on the co-discoverers of the theory of natural selection in organic evolution, viz., Darwin and Wallace (1858). An interesting review of the historical evolution of the connection between economy and ecology can be found in Rosser (1991) where also the 'dialectical' difficulties between these disciplines are pointed out.

In this context it is interesting to recall that two 'ecological' models can be considered to be the main 'sources' of many subsequent models applied in economic-social sciences, i.e. the Lotka-Volterra model and the May model.

a) The Lotka-Volterra Model

The Lotka-Volterra model describes, by means of non-linear differential equations, the cyclical evolution of two species: one species, the prey, is restrained in its growth by the presence of a predator, which feeds on it; the other one, the predator, is positively related to the prey population (see Lotka, 1920 and Volterra, 1931). The work of Lotka-Volterra was essentially recognized in economic sciences only in 1967 by Goodwin, who used this concept for describing both the motion of the employment rate and that of workers' income share. Further applications were then related to population and gross domestic product (Dendrinos and Mullally, 1985), population and land price (Orishimo, 1987), transport flows and workplaces (Nijkamp and Reggiani, 1990), market evolution (Curry, 1981) and innovation-diffusion processes (Camagni, 1985, Nelson and Winter, 1982, and Sonis, 1986).

b) The May Model

May's model is a simple form of the S-shaped logistic equation (first developed by Verhulst, 1838) in which a time lag of one generation exists (in other words, the logistic function assumes a discrete form). May's model has already been discussed in its theoretical foundation in Section 3 and - in its empirical application - in Section 4. We aim to show here how the May model and the Lotka-Volterra model can be integrated in the broad concept of a **niche**.

6.2 Niche theory: a unifying approach

Niche theory has become a popular concept in ecology and biology starting from Grinnell (1917) who used the term 'niche' in order to describe 'the functional role and position of an organism in its community'. Later on the concept of niche has been used in a wide variety of different contexts (see for a review Nijkamp and Reggiani, 1992b). Following Pianka (1978, p. 238), we will now define here a niche as "the total sum of the adaptations of an organismic unit or as all of the various ways in which a given organisms unit conforms to its particular environment". This definition emphasizes in particular dynamic feedback patterns, which are the subject matter of this section.

Recently, the niche concept has also been linked to the phenomenon of inter-species competition and to dynamic patterns of resource utilization.

The formalization of the niche concept intends to express optimal adjustment (or survival) processes in dynamic systems with scarce resources. Usually, niche relationships among potentially competing species are often visualized by means of tolerance curves (which are typically bell-shaped and unimodal) with their peaks representing **optimal conditions** for a particular process and their tails the limits of tolerance. In other words the niche concept can be formalized by a utilization function (of a species) against a resource spectrum. A central aspect of niche theory concerns then the amount of

resource sharing, or **niche overlap**, in a chain of niches.

As a starting point we will analyze here the prototype model of several competing populations studied by Lotka and Volterra and interpreted on the basis of niche theory by May (1973):

$$\dot{x}_i = x_i (k_i - \sum_{j=1}^m \alpha_{ij} x_j) \quad (8)$$

where x_i is the population of a species i ($i = 1, 2, \dots, m$), the constant k_i represents the suitability of the environment for the i th species (e.g., carrying capacity) and the competition coefficients α_{ij} measure the niche overlap in the utilization functions. It is clear that many particular cases can arise from the base equation (8). For example, for $i=1$, the well-known logistic function emerges (and consequently the May equation in its discrete form). Clearly, when the parameters vary, we get a biological or ecological evolution, as described by Prigogine and Stengers (1984, pp. 193-194) as follows: "Living societies continually introduce new ways of exploiting resources or of discovering new ones (that is K increases) and continually discover new ways of extending their lives or of multiplying more quickly. Each ecological equilibrium defined by the logistic equation is thus only temporary, and a logistically defined niche will be occupied successively by a series of species, each capable of ousting the preceding one when its 'aptitude'¹⁴ for exploiting the niche, becomes greater" (See Figure 4).

In this framework, system (8) may be applied to socio-cultural and economic evolution (where the population dynamics can be extended to urban and regional development, economic activities, diffusion of ideas, transport systems, etc.) in which learning mechanisms, innovations or technological changes exist. In other words, we are then facing a choice situation with different strategies which can be adopted or rejected by surrounding 'populations'.

Equations based on formulation (8) have been applied, for example, to urban dynamics (see Allen and Sanglier, 1981 and Camagni et al. 1985) where each center's growth path is subject to successive bifurcations which are linked to the appearance of new economic functions as well as to the pace of general technical progress. In general, an evolutionary model of type (8) can be interpreted in the framework of the self-organization of systems (i.e., the inner dynamics which drive them to reconstitute themselves in new structures (see Prigogine, 1976)), or in the framework of self-renewal (or self-production) of systems; in other words system (8) is also an autopoietic system¹⁵.

After these introductory observations, the interesting question arises whether system (8) - if expressed in a discrete form - may generate also a chaotic evolution, and if so, under which conditions. Therefore, in the next sub-section we will analyze an illustrative case of two competing niches and examine in particular their evolution in both continuous and discrete time.

Figure 4 about here

¹⁴ It is clear that this 'aptitude' is measured by obtaining x_i obtained by the equilibrium condition $\dot{x}_i = 0$.

¹⁵ Autopoietic organization can be defined as "a network of interrelated components - producing processes such that the components, through their interaction, generate recursively the same network of processes as an identifiable unity in the space in which the components exist. The product of an autopoietic system is necessarily always the system itself, its organization being continuously realized under permanent turnover of matter and energy" (see Zeleny and Pierre, 1976, p. 7).

6.3 Spatial competition and Ecologically-Based Model: An Illustration

6.3.1 Introduction

For the sake of illustration we will provide here a simple example and related numerical experiments regarding a model for spatial competition based on elements from population dynamics. Suppose the existence of two cities oriented towards the same product market, viz., high tech products. One city is a major centre which aims to build up a general profile of a technopolis with a broad range of high quality segments such as micro-electronics, bio-technology, telematics, etc. The other city is much smaller and is only able to focus on one high tech segment, for instance, biotechnology. It does not have the critical mass to build up a really significant growth pole in this area, although it is able to have a self-organized stable equilibrium. If the large city - due to its synergy (economies of scale and scope) - is growing as a self-sustained technological centre, it may attract well-trained employees not only from within its territory, but also from other places (including the smaller second centre). Thus the growth of the larger city is detrimental to that of the smaller place, whereas the opposite does not - or hardly - take place. Such phenomena - well-known in spatial competition - can easily be described by means of ecologically-based models, while also their stability properties can be investigated by using the above notion from the theory of chaos.

6.3.2 A hierarchical model of two niches for a spatial system

In this subsection we will show how the above mentioned situation, reflecting a certain kind of hierarchy in a spatial system, can be modelled by means of niche theory.

In general, in the case of two competing systems, equation (8) - in continuous form - results as follows (for $x_1 = x$ and $x_2 = y$):

$$\begin{aligned}\dot{x} &= x(a - bx - cy) \\ \dot{y} &= y(d - ey - fx)\end{aligned}\quad (9)$$

where the parameters a and d represent the carrying capacities of x and y respectively, b and e are the related growth rates and c and f are the competition coefficients which measure the niche overlap.

Stability conditions of system (9) are well known (see Smith, 1974). In particular it can be shown that if the carrying capacities a and d are not equal, the coexistence of the two species x and y likely holds by reaching a stable equilibrium. However, if the two species have identical requirements, the most efficient species will eliminate their competitors (see, for details, also Nijkamp and Reggiani, 1992b).

It is interesting to note that if system (9) is expressed in discrete terms as follows:

$$\begin{aligned}x_{t+1} &= x_t(a - bx_t - cy_t) \\ y_{t+1} &= y_t(d - ey_t - fx_t)\end{aligned}\quad (10)$$

competitive interaction does again not produce oscillations (see for a proof, Smith, 1974).

Let us now consider the case of a competitive hierarchical structure in a spatial system as described in Section 6.3.1. In this hypothesis system (10) can be reduced to:

$$\begin{aligned}x_{t+1} &= x_t(a - b)x_t \\ y_{t+1} &= y_t(d - ey_t - fx_t)\end{aligned}\quad (11)$$

where x represents the size of the large centre and y the size of the sub-centre.

System (11) represents now the impact of species x , whose evolution follows the logistic equation of a May type, on species y , without feedbacks effects. The equilibrium analysis regarding system (11) shows the existence of two fixed points, a trivial one $A(0,0)$ and a non-trivial one $B[(a-1)/a; (ad - a f + f - a)/ae]$.

For the second fixed point B it is easy to find the conditions for which a Hopf bifurcation occurs by following the Ruelle-Takens theorem (see, for example, Lorenz, 1989), viz.:

$$d^* = (a^2f + 2a^2 - 3a - 3af + 2f) / (a^2 - 2a) \quad (12)$$

Condition (12) implies that when the carrying capacity of centre y exceeds the critical value d^* , the fixed point B becomes unstable with the possibility of oscillations.

This result is indeed remarkable, since it shows the possibility of oscillations - in a system which is in itself not oscillatory - as soon as the evolution of the dominant centre follows a chaotic pattern. Simulation experiments related to result (12) will be offered in the next subsection.

6.3.3 Simulation experiments

In this subsection we will investigate the behaviour of system (11) before and after reaching the critical value d^* leading to a Hopf bifurcation (see condition (12)).

In particular we will consider values of the parameter a displaying in the May equation both irregular (for example, $a=3.6$) and chaotic behaviour (for example, $a=3.9$). Here we will consider two cases:

$$d < d^* \quad \text{and} \quad d > d^*$$

In the first simulation (see Figure 5) we will assume the following parameter values:

$$a = 3.6 ; d = 1.5 < d^* ; e = 1 ; f = 0.8$$

with the initial condition

$$x = y = 0.1 ,$$

while for the second simulation (see Figure 6) we will assume:

$$a = 3.9$$

by keeping the other parameter values constant.

Figure 5 shows an oscillatory pattern for variable x (as can easily be understood from the value of a) and stability in the long run for variable y, owing to the fact that we kept the growth parameter d below the critical value d^* . The same situation happens in Figure 6. Even though centre x shows a chaotic pattern, the competing centre y reaches stability in the long run; only in the short run y is more oscillatory than in the previous case.

It is interesting now to observe the stabilization of centre y when we consider a value of the growth parameter d beyond the critical value d^* .

Figures 5, 6, 7 and 8 about here

In particular, Figure 7 shows an irregular behaviour in the evolution of both centres x and y for the following parameter values:

$$a = 3.6 ; d = 3.3 > d^* ; e = 1 ; f = 0.8$$

while Figure 8 displays completely a even more irregular pattern, due to the 'chaotic' value of a ($a = 3.9$) (while keeping the other parameter values constant).

6.4 Concluding remarks

In this section we have shown how ecologically-based models and in particular niche theory may offer interesting insights into the analysis of the evolution of spatial economic systems. In particular, the niche concept seems very useful to illustrate the evolution of a self-organizing system, in which the ecological fluctuations are represented by new competitors.

In this context the analysis of a particular case of a hierarchical system of two niches - applied to urban development - has shown the relevance of a 'chaotic' evolution. It appeared in fact that in the presence of a chaotic evolution for the dominant centre - together with the conditions for the onset of cycles (i.e. the conditions for the existence of a Hopf bifurcation) - also the evolution of the competing centre becomes irregular or chaotic.

7 General Conclusions

In this paper we have shown how the discovery of 'chaos' seems to have created a new paradigm in scientific modelling which generates intriguing research questions. Firstly, the process of verifying theories on dynamic systems behaviour through conventional predictions becomes more problematic in case of chaotic systems. And secondly, the concept of chaos demonstrates that a system can have a complex global behaviour at large which in general cannot be deduced from knowledge on its constituent parts. It is interesting to quote here Crutchfield et al. (1986, p. 57) who claim even "chaos provides a mechanism that allows for free will within a world governed by deterministic laws".

This points out a final important issue, viz., the question of the **overall stability of a system's model**, if one of the subsystems is governed by chaotic motion. An example has been in the penultimate section, where the impact of the logistic growth of a May type has been studied in a broader system of the competing dynamics of an ecologically-based model. In this context it should be noted that even though bifurcation, catastrophe and chaos theory have become components of a new framework for investigating the long term evolution of spatial economic systems, it is also evident that still a long trajectory has to be followed before the 'new dynamics' movement will have led to operational and testable analytical propositions which can also be used in empirical research. For this purpose various new research directions are necessary in order to complement the tools developed so far in dynamic analysis of spatial economic systems.

A systematic listing of such new tools suggests the design of a research agenda with the following central items:

1. **Specification theory.** The formulation of dynamic economic systems models which are compatible with plausible behavioural hypotheses on the one hand and which lend themselves for empirical testing on the other hand is a difficult methodological task which so far has not yet been very successful. The extent to which a dynamic economic model is a satisfactory mapping of highly dynamic real world processes is a formidable research effort, mainly since empirical tests are lacking.
2. **Verification analysis.** The question whether (theoretical) model results are in agreement - in a qualitative or quantitative sense - with non-linear patterns incorporated by an underlying data set is another important research challenge. So far it has been very difficult to find statistically satisfactory parameter estimates in non-linear models whose value at the outset is falling in the chaotic domain. Besides, the statistical tools for identifying non-linear dynamic (and possibly chaotic) patterns are not very well developed, although the value of the Liapunov exponents, the Brock-Dechert-Scheinkman test, and the use of recurrence plots may provide analytical support.
3. **Behavioural analysis.** The identification of chaos behaviour in the decisions of actors is to a large extent dependent on the degree of aggregation of observed time series. In a very short time span the possibility of chaotic patterns in behaviour is much higher than in a longer time span, as in the latter case a smoothing amendment may take

place. Furthermore, in the longer run rational expectations of actors would generate negative feedback reactions, so that wild fluctuations would be prevented. Altogether, it is difficult to separate random shocks, measurement errors, impacts of time series and behavioural feedbacks in a given data set.

4. **Impacts of time delays.** Although it was sometimes assumed in the past that the inclusion of more time lags would destabilize a growth trajectory, it has recently been recognized that this is not necessarily true (e.g., in the rational expectations model the probability of occurrence of chaos diminishes if the weight of the past increases). In recent publications it has been demonstrated that an increase in time lags may increase the probability of chaos, but on a much smaller domain. Two research directions might be interesting in this context, viz. the relevance of fractal theory (which takes for granted that phenomena at a given level are replicated at lower levels) and of percolation theory (which analyzes the time trajectory of a dynamic phenomenon in case of unstructured barriers).

5. **Impact of chaos modules.** This question focuses on the overall stability of a non-linear dynamic systems model, if this model incorporates one smaller module which may exhibit chaotic behaviour. This leads to the intriguing research question whether lower order chaos may affect overall stability and vice versa. In this context there is much scope for continuing innovative research strategies, viz. **niche theory** (which deals with partly overlapping and interwoven sets of populations in a dynamic system) and **autopoiesis** (which addresses the issue of self-organization in dynamic social-cybernetic or self-organizing systems).

It is clear that in all above research suggestions the behavioural aspects of actors are of decisive importance. This once more emphasizes the need for an integration of behavioural modelling with meso/macro dynamic spatial economic modelling. In this context - and given the usual lack of appropriate time series data - the recent trend towards **experimental social science research** is undoubtedly an important step forward.

In the specific context of chaos theory another issue should also be noted concerning the study of mechanisms through which fluctuations may be **spatially** amplified. This implies, on the one hand, investigation of **spatial autocorrelation** patterns related to chaos emerging in a development process of a spatial system; on the other hand, it would imply further analysis of non-linear lattice dynamics, i.e. network systems that are discrete in time and space, but with continuous state variables.

In light of the above mentioned research issues, a further aspect which deserves to be investigated is the analysis of **spatio-temporal intermittency**, which includes a spatial extension to temporal behaviour. In other words, it is worth examining whether local dynamics, in combination with spatial diffusion, can lead to turbulent regions that intermittently form complex space-time structures.

In conclusion, the study of **spatio-temporal chaos** may generate a new perspective in spatial dynamics, from both a theoretical and topological point of view, as well as a new understanding of many complex phenomena.

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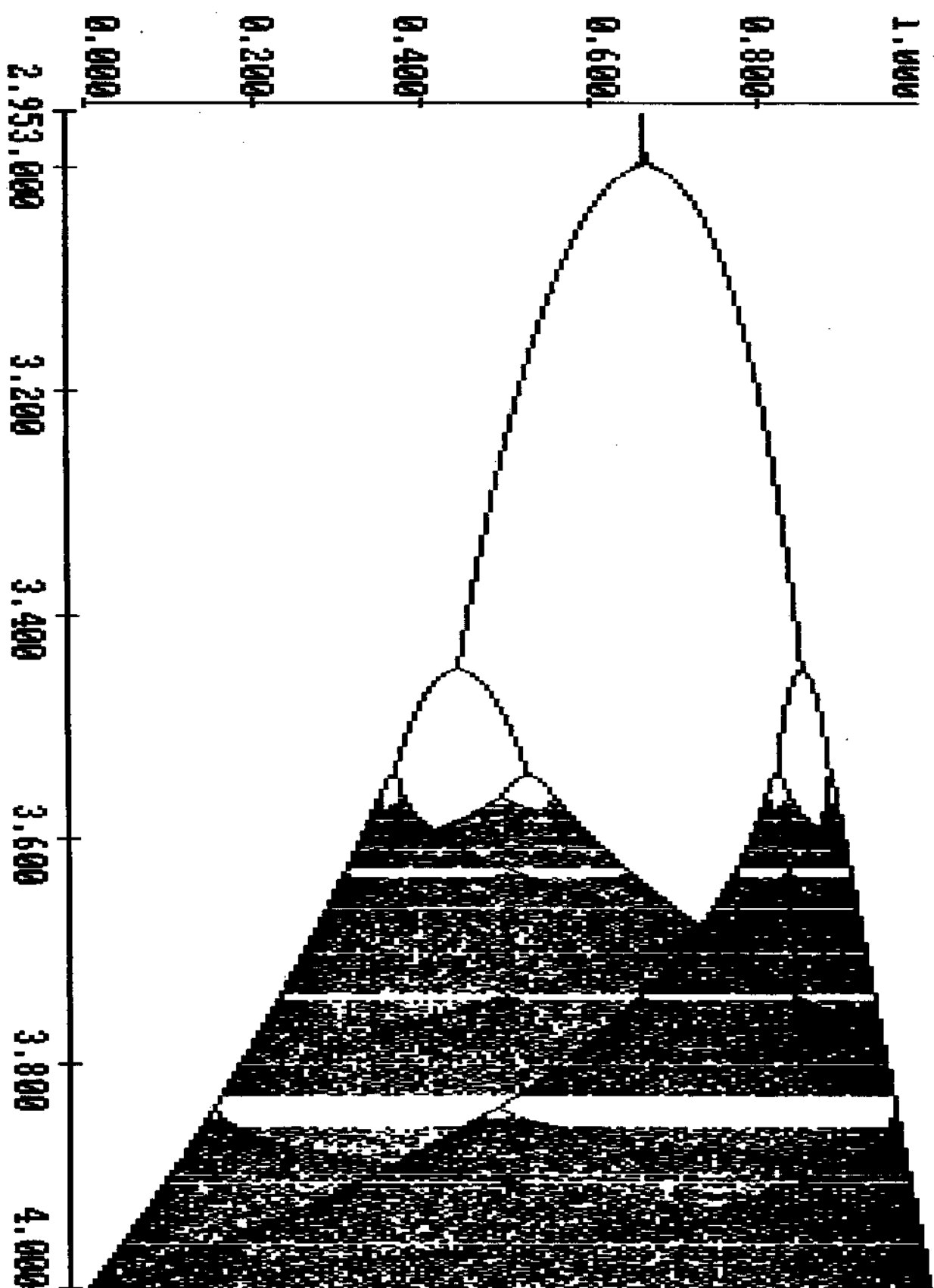


FIGURE 1. Bifurcation Diagram for the Logistic Map

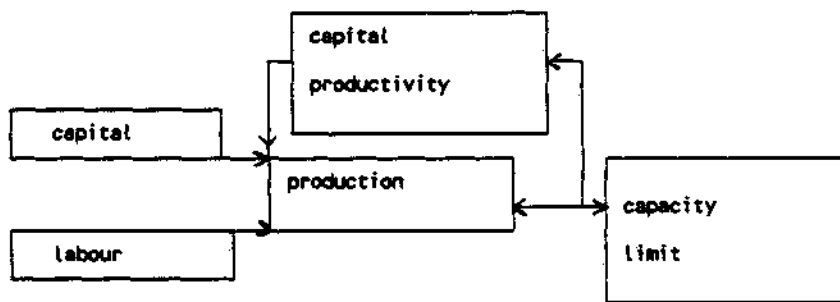
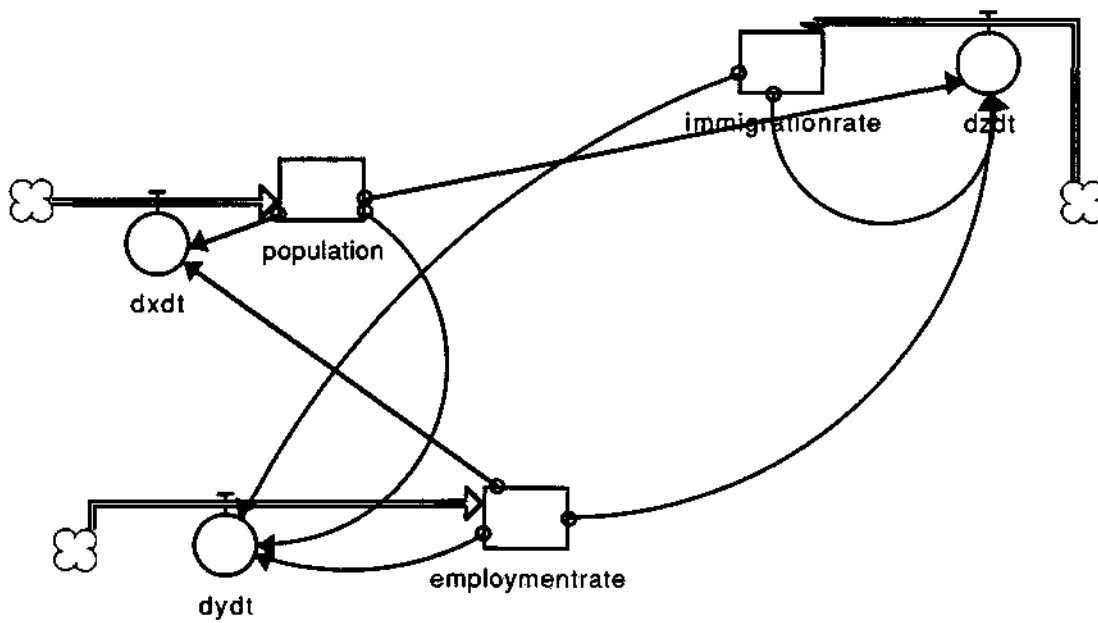


FIGURE 2. A Simple Production System with Exogenous Forcing Limits



**Figure 3. A Diagrammatic Representation in Stella-
Format of an Urban Dynamic Model.**

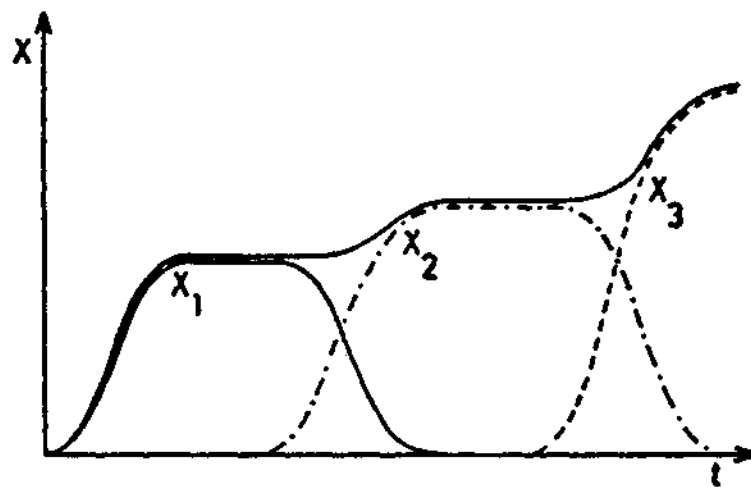


FIGURE 4. A Chain of Niches

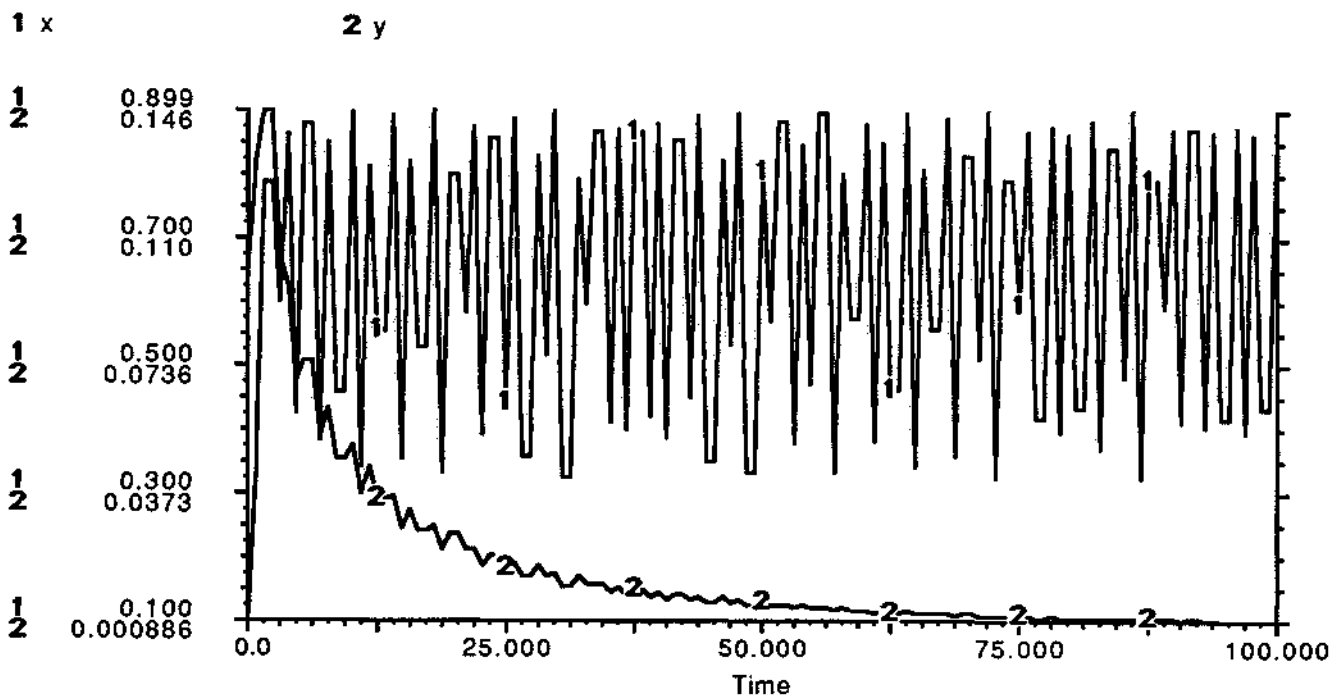


FIGURE 5. Oscillatory Behaviour for Centre x and Stable Behaviour (in the Long Run) for Centre y for $d < d^*$ and $a = 3.6$

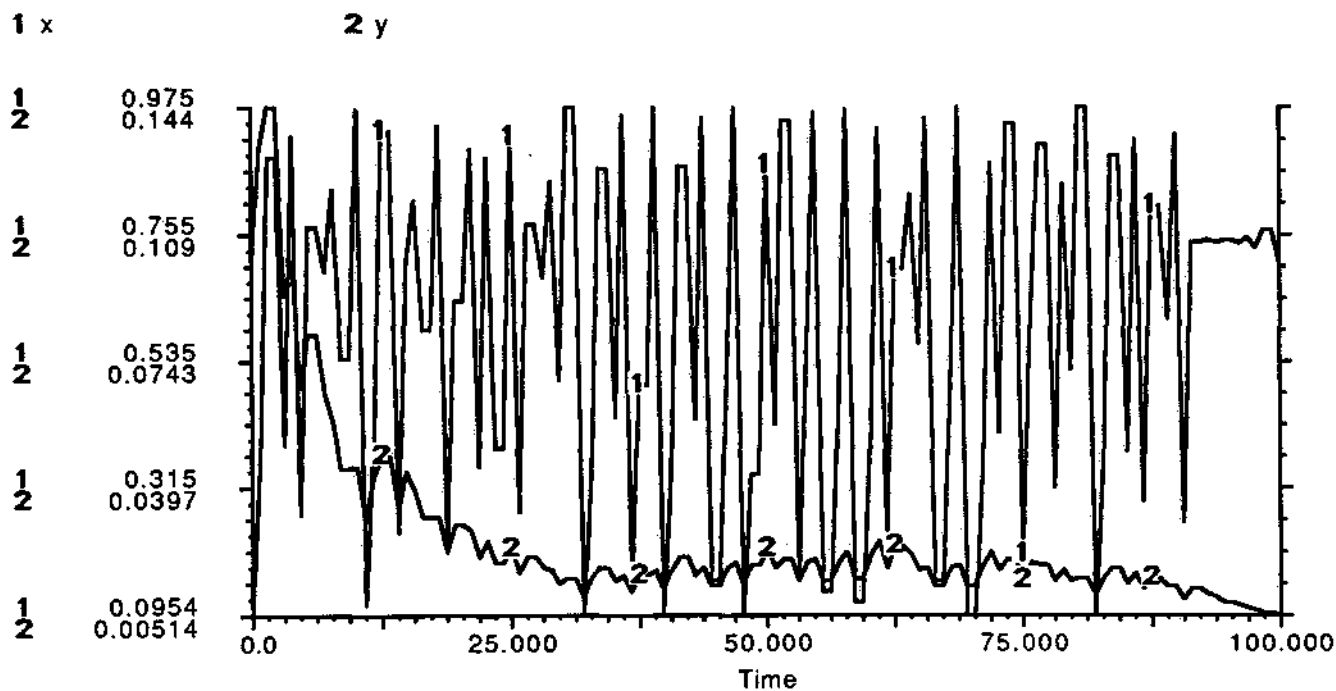


FIGURE 6. Chaotic Behaviour for Centre x and Stable Behaviour (in the Long Run) for Centre y for $d < d^*$ and $a = 3.9$

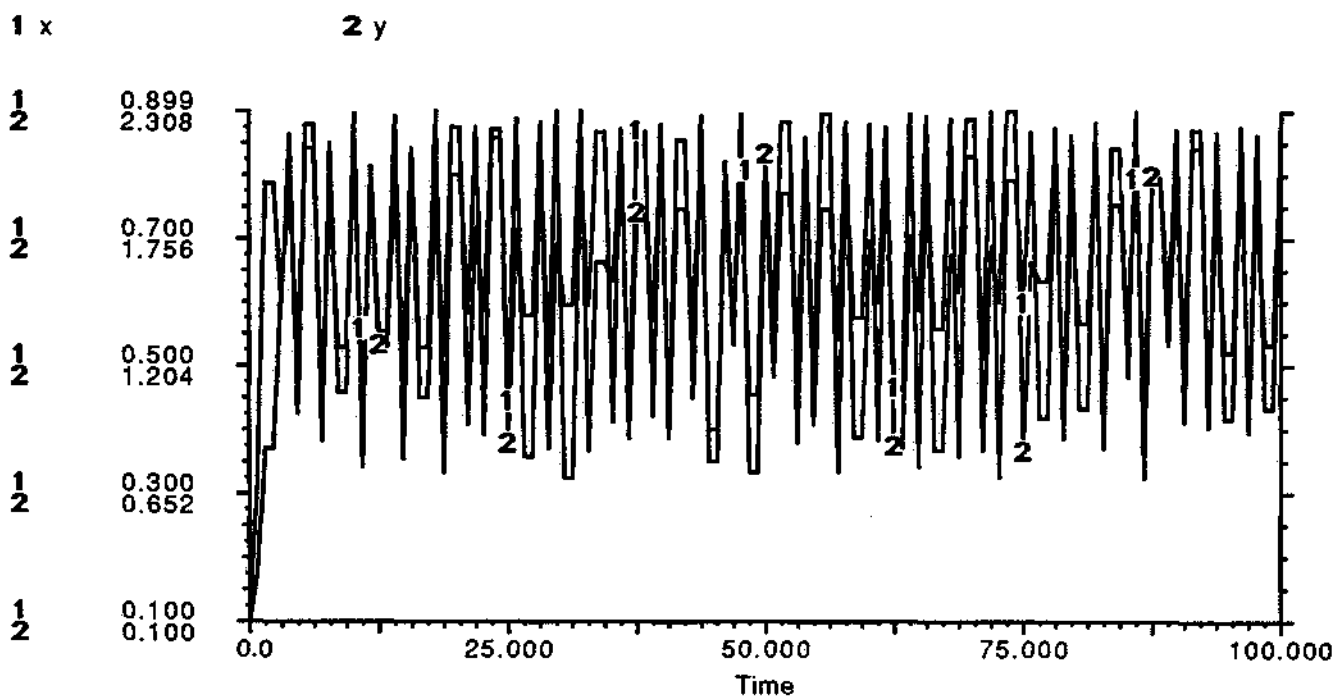


FIGURE 7. Irregular Behaviour for both Centres x and y, for $d > d^*$ and $a = 3.6$

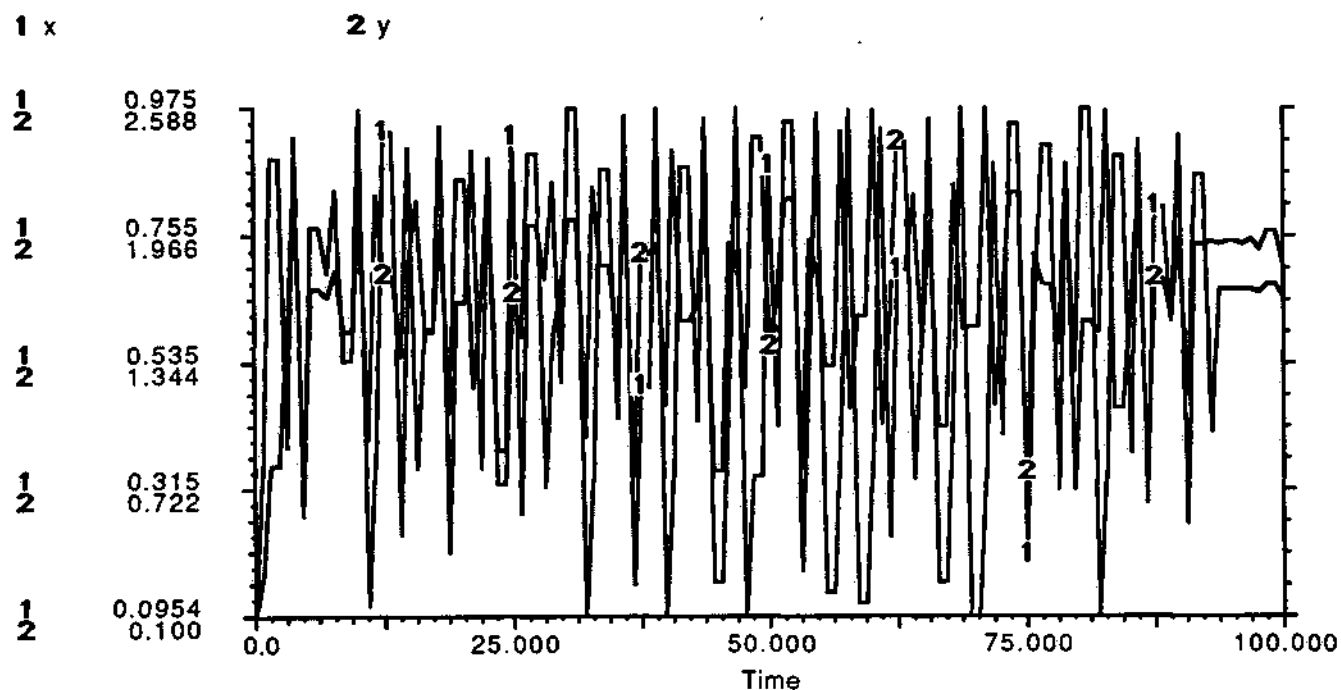


FIGURE 8. Chaotic Behaviour for both Centres x and y, for $d > d^*$ and $a = 3.9$.

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